

VIGNAN'S INSTITUTE OF MANAGEMENT AND TECHNOLOGY FOR WOMEN

(An Autonomous Institution)

I-B.Tech.-I-Semester Regular Examinations, February-2025

MATRICES AND CALCULUS

(CSE)

Time: 3 Hours

Max. Marks: 60

(Answer All Questions)

Note: Question paper consists of Part-A & Part-B.

- **Part-A** for 10M, ii) **Part-B** for 50marks
- **Part A** is compulsory, consists of 10 sub questions from all units carrying equal marks.
- **Part-B** consists of **10 questions** (numbered from 2 to 11) carrying **10marks** each. From each unit there are 2 questions and the students should answer one of them. Hence the student should answer **5 questions** from **Part-B**.

PART- A

(10Marks)

- 1.a) Define Echelon form of a Matrix 1M
- 1.b) What is the rank of a unitary matrix of order 'n' 1M
- 1.c) If the Eigen values of A are 1,1,2 then find the Eigen values of A^3 1M
- 1.d) State Cayley-Hamilton Theorem. 1M
- 1.e) Discuss the applicability of Rolle's mean value theorem for the function $f(x) = |x|$ in $[-1, 1]$ 1M
- 1.f) Define Gamma function 1M
- 1.g) If $u = \frac{y}{x}, v = xy$ then find $J\left(\frac{u,v}{x,y}\right)$ 1M
- 1.h) Write the necessary conditions for f (x, y) to have a maximum or minimum at (a, b). 1M
- 1.i) Evaluate $\int_{x=1}^3 \int_{y=0}^1 xy^2 dy dx$ 1M
- 1.j) Sketch the area bounded by the curves $y = x, y = x^2$ 1M

PART-B

(Answer any one full question from each unit. Each question carries 10 marks)

(50Marks)

2. a) Find the rank of the matrix by reducing it to Normal form $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$ 5M
- b) Investigating for what values of λ and μ do the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have (i) no solution (ii) a unique solution (iii) An infinite number of solutions. 5M

OR

3. a) Find the rank of a matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 4 & 8 & 7 & 5 \end{bmatrix}$ by reducing it to Echelon form. 5M
- b) Use Gauss Seidel iteration method to solve the system $10x + y + z = 12$, $2x + 10y + z = 13$, $2x + 2y + 10z = 14$. 5M
4. a) Prove that if λ is an Eigen value of a non- singular matrix A then $\frac{|A|}{\lambda}$ is an eigen value of $\text{Adj}A$. 2M
- b) Reduce $x^2 - y^2 + 4z^2 + 4xy + 6xz + 2yz$ into canonical form by orthogonal transformations and also find its rank, index and signature 8M

OR

5. a)

Find Eigen values and Eigen vectors of $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

5M

b) Verify Cayley-Hamilton theorem and hence find A^{-1} where

$$A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

5M

6. a)

Verify Rolle's theorem for $f(x) = \log \left[\frac{x^2 + ab}{x(a+b)} \right]$ in $[a, b]$

5M

b) Show that $\Gamma(1/2) = \sqrt{\pi}$

5M

OR

7. a) Show that $1 + x < e^x < 1 + xe^x$, for $x > 0$

5M

b)

Show that $\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ where n is positive integer and $m > -1$

5M

8. a) Prove that $u = x + y + z$, $v = xy + yz + zx$, $w = x^2 + y^2 + z^2$ are functionally dependent and find the relation between them

5M

b) A rectangular box open at the top is to have a volume 32 cubic feet. Find the dimensions of the box requiring least material for its construction

5M

OR

9. a) If $u = \frac{x+y}{1-xy}$, $v = \tan^{-1}x + \tan^{-1}y$ then prove that u, v are functionally dependent and also write relation between them

5M

b) Find three positive numbers whose sum is 100 and whose product is maximum

5M

10. a) Change the order of integration $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy dx$

5M

b) Evaluate $\iint_R r^3 dr d\theta$ Over the area included between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$

5M

OR

11. a) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dx dy$

5M

b) Evaluate $\iiint_V (xy + yz + zx) dx dy dz$. Where V is the region of space bounded by the planes $x=0$, $x=1$, $y=0$, $y=2$, $z=0$ and $z=3$

5M

*****VMTW*****